Proceedings of the American Academy of Arts and Sciences.

Vol. XLII. No. 6. — July, 1906.

CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY,
HARVARD COLLEGE.

FRICTION AND FORCE DUE TO TRANSPIRATION AS DEPENDENT ON PRESSURE IN GASES.

By J. L. Hogg.

CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY OF HARVARD COLLEGE.

FRICTION AND FORCE DUE TO TRANSPIRATION AS DEPENDENT ON PRESSURE IN GASES.

By J. L. Hogg.

Presented by J. Trowbridge, November 8, 1905. Received April 6, 1906.

Problem in General and Results in General.

This is a preliminary paper on an investigation whose object is threefold.

It is known that at pressures lower than about 1 cm. of mercury the resistance which a solid body meets in passing through a gas is noticeably smaller than it is when the pressure is that of an atmosphere. This is because the phenomenon of slip presents itself at the low pressures. The first object of the investigation is to study fully the relation between the friction of a gas on a solid body and the pressure in the gas at those pressures where slip is appreciable.

Again, it is known that, if a given volume of gas is divided into two portions by means of a partition one side of which is kept hotter than the other, and if a small opening of any form be made in the partition, the gas will flow from the cold side to the hot side of the partition until there is an excess of pressure in the chamber contiguous to the latter. This excess of pressure depends on the difference of temperature of the two sides of the partition and on the mean pressure.

The second object is to examine in a particular simple case the relation between the force on the partition (called the transpiration force), resulting from this flow of gas towards the hot side of the partition, and the mean pressure in the gas.

The third object of the investigation is to examine the feasibility of measuring the pressure in a gas when either of these relations is known without having recourse to the McLeod gauge. That this third object is by no means the least significant will appear when the evidence as to whether the McLeod gauge is reliable for the measurement of small pressures is adduced.

The progress made in the solution of these problems up to the present may be summed up as follows:

(1) The great difficulty in devising and carrying out methods of constructing the pieces of apparatus required has been overcome. This, which was by no means the least difficulty encountered, can be appreciated only when one knows the form of the apparatus required, and the treatment which the apparatus must receive in order that all traces of moisture may be removed from all the solid surfaces exposed to the interior of the containing vessel.

(2) The friction has been measured at pressures ranging from that of an atmosphere to 0.00024 mm. of mercury, the small pressures being measured by the McLeod gauge. Figure 6, in which ordinates are proportional to friction and abscissas proportional to pressure, shows how the friction diminishes as the pressure diminishes. The curve, whose error is certainly less than one per cent (if the McLeod gauge can be considered trustworthy over the range of pressures indicated), is very regular; and a discussion of the numerical results in that part of the curve which corresponds to pressures above 0.1 mm. of mercury shows that the law

$$\left[\frac{L-K}{l-K}-1\right]p=c,$$

first deduced by Kundt and Warburg for pressures down to 0.6 mm., holds very well down to about 0.1 mm. Beyond this point, however, the theoretical curve, i. e., the curve obtained by calculating p from the above relation after the constants have been determined from observations at comparatively large pressures, does not coincide with the actual curve.

(3) The force due to transpiration has been measured over a range of pressure varying from 1.42 mm. to 0.0093 mm.

(4) Figure 7 (the error in the curve may very well be five per cent), in which the ordinates are proportional to the transpiration force and abscissas to pressure, shows that, for pressures below that for which the force is a maximum, the relation between the force and the pressure rapidly approaches a relation of mere proportionality, or at worst a proportionality disturbed by a constant term. In other words, the curve becomes a straight line passing very nearly through the origin.

Problem in Detail.

Introduction.

Aside from the interest which attaches itself to an investigation of the law relating viscosity to pressure at low pressures, the advantage of being able to measure gas pressure by measuring the friction of the gas on a pendulum is apparent. By this method the state of the gas remains unchanged during the time the measurement is in progress, while by the method of the McLeod gauge the gas must be compressed in order that the pressure may be measured. It is, however, beyond doubt very important that in establishing a method of gas pressure measurement by friction, or otherwise, a second method should if possible be obtained to serve as a check upon the one which we may consider most convenient and therefore the most desirable. Though the investigation of the law relating pressure in a gas to transpiration force when the transpiration space is of special form is interesting and important for its own sake, yet additional interest and importance is imparted to it when the possibility of making its results serve as the desired check upon the proposed friction method of measuring gas pressure is considered.

Thus, in so far as the object of the present investigation is to test or replace the McLeod gauge, the function of what has been called the transpiration apparatus is to serve as a check upon the results of pressure measurement obtained by the viscosity apparatus. As will appear in the sequel, the problem to be solved with the transpiration apparatus is that of investigating the law of gas action on a radiometer vane of special form surrounded by a containing vessel whose parts are symmetrically placed with respect to the vane.

That at least another attempt should be made to decide the question as to how much reliance can be placed on the McLeod gauge for measurement of pressures below 0.1 cm. is very forcibly brought home to one who examines the conflicting evidence of high authority.

The attempts to investigate the behavior of the gauge when the vacua are high, have as a rule been made, not with the avowed intention of testing the gauge, but with the object of testing the validity of Boyle's law at various pressures, any departure from which for any gas would at once limit the use of the gauge for that particular substance. The method used was to enclose a certain quantity of gas, measure accurately its volume and its pressure, calculate $p v$; change this volume, and therefore pressure; measure again, and so on. The various values of $p v$ thus obtained should be the same, that Boyle's law may be obeyed, and hence that the principle of the gauge may be sound.

Siljeström[1] found departures from the law in the case of air and oxygen. Amagat[2] found the law obeyed, while Bohr[3] found an anomaly in the case of oxygen at 0.7 mm. pressure. In the curve representing $p\,v$ plotted against p there is a sudden drop at this pressure. Batelli's[4] experience corresponds well with that of Bohr. At 0.7 mm. with oxygen there is departure, while with air the departure is at pressures between 2 mm. and 5 mm. Carbon dioxide departs from the law, while hydrogen is found to obey it from one atmosphere to 0.002 mm. Baly and Ramsay[5] have pronounced the gauge worthless for oxygen, while the results given for hydrogen are said to be reliable.

Lord Rayleigh,[6] working with values of p ranging from 1.5 mm. of mercury to 0.01 mm., found no evidence of any anomaly with oxygen, nitrogen, or hydrogen; while at pressures ranging from 75 mm. to 150 mm. he found the law fully obeyed.

There is, then, conflicting evidence, with much in favor of irregularity.

Although with the gauge which Rayleigh employed in his investigation one can measure, with an accuracy of about five per cent, pressures of 0.01 mm. of mercury, yet for pressures which in many cases must be measured, this method is not sensitive enough, and then one is forced back upon the use of the McLeod gauge, and that in a region where we know nothing of its action.

We are indebted to Sutherland[7] for the suggestion that the lowest pressures may possibly be measured by measuring the friction of the gas on a pendulum at pressures where the phenomenon of slip can be detected experimentally. Having had some experience in determining the absolute value of viscosity in gases, I have undertaken to investigate the whole subject.

As has been stated at the beginning of this paper, the problem involves the solution of two others, which will now be discussed in detail.

The Friction Problem.

When a gas flows over a solid surface at a uniform rate, or a solid surface is made to move through a gas with uniform velocity, there is brought into play a resistance to the motion due to friction. If, for example, we have two parallel planes placed in a mass of gas at, say, unit distance apart, and, while keeping one fixed, we cause the other to move in a certain direction in its own plane, a certain force must be

---

[1] Pogg. Ann., **151**, 1874.

[2] Ann. de Chim. et Phys., **28**, 1883.

[3] Wied. Ann., **27**, 1886.

[4] Phys. Zeits., **3**, 1901.

[5] Phil. Mag., **38**, 1894.

[6] Phil. Trans., **196**, 1901.

[7] Phil. Mag., [5], **43**, 1897.

continually applied in the direction of motion in order that a uniform velocity may be maintained. That is, during the motion there is a certain tangential stress exerted by the gas on the solid which opposes the motion of the solid. If the layer of gas which is in contact with the moving plane moves with the same velocity with which the solid moves, the tangential stress between the gas and the solid, that is, the resistance experienced by the solid, must depend upon the force required to cause one layer of the gas to move with a certain velocity relative to the next layer. Experiment shows that the relative velocity of contiguous layers is a measure of the tangential force. If, however, the solid and the gas in contact with it do not move together, then, for the same velocity of the moving solid, the velocity of a layer of gas relative to its contiguous layer is less than before, and hence the friction between them is less, and therefore the stress at the solid is less. In the case considered, when the moving plane and the layer of gas next to it move together the force per square centimeter of the plane necessary to keep up a uniform velocity of one centimeter per second in the given direction is the coefficient of viscosity or internal friction. If the moving plane and the layer of gas next to it have not the same velocity, the gas is said to slip on the solid. The force per unit area of the plane which must be applied in a given direction in the plane to maintain a uniform relative velocity between the plane and the layer of gas is proportional to this relative velocity. When the relative velocity is one centimeter per second the force required is the coefficient of external friction.

Maxwell showed that the internal friction of a gas is constant for pressures varying from atmospheric to one sixtieth of atmospheric pressure. In his paper [8] he defines the coefficient of slip to be the ratio μ/σ, where μ is the coefficient of viscosity of the gas, and σ the coefficient of external friction. The coefficient of slip is then a quantity which decreases as σ increases and μ decreases, and which increases as σ decreases and μ increases. In order that there should be no slipping, σ must be infinitely great compared with μ. As this is probably never the case, there is some slip under all circumstances. Maxwell also showed that when the conditions are such that slip must be considered, the resistance to the moving solid in the foregoing discussion is the same as it would be were the fixed surface removed a distance $2\,\beta$ farther from the moving surface where β is the coefficient of slip.

A consideration of Figure 1 will probably help to make the motion of the gas between the planes better understood in the case where slip

[8] Sci. Papers, **2**, 1.

must be considered as well as in the case where it is negligible. Let A B represent the section of the fixed plane just considered by the plane of the paper. Likewise, let C D represent the similar section of the moving plane. Let the moving plane have a velocity of one centimeter per second in the direction C D, and let the distance between the planes be one centimeter. If the layer of gas next to the solid moves with the same velocity with which the solid moves, the line of particles represented by X Y at the beginning of any second will, if X O equals X Y, be represented by X O at the end of this second. That the particles will at the end of the second lie in the line X O is seen by considering the forces acting on any layer of the gas between the planes and parallel to them. The layer above the one in question exerts a tangential force on the latter which tends to move it in the direction C D, but the layer below it holds it back with a force just equal to this. Otherwise there would be a net constant force acting on the layer parallel to C D, which would produce an acceleration. Clearly there can be no acceleration of the layer while the moving plane maintains its uniform velocity. Moreover, since action and reaction are equal, the layer which is thus holding back the layer above it is being dragged forward with the same force. It in turn exerts the same forward force on the next succeeding layer, and so on. It thus appears that the force exerted by the layer of gas next to the fixed solid, A B, in the direction A B, is just the same and in the same direction as that exerted by any layer of gas upon that layer just beneath it; and also that the force with which the fixed solid resists the motion of the gas is the same as the force with which any layer resists the motion of the layer above it. Clearly, a similar statement may be

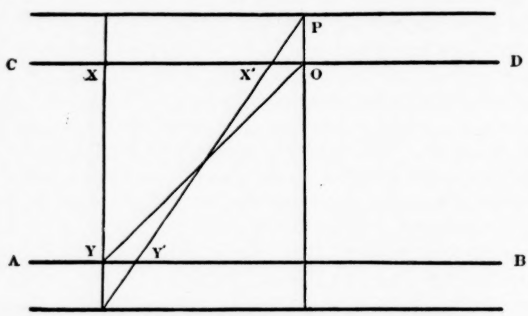


FIGURE 1.

made regarding the resistance which the gas in contact with the solid, C D, offers to the motion of C D. Under these circumstances, in any second, any one layer must move with respect to the next layer below it exactly the same distance which the one above it moves with respect to the layer in question. It follows, then, that the line of particles, X Y, at the beginning of a second will become the line X O at the end of that second.

If, however, the solid and the gas in contact with it have different velocities, that is, if the gas slips on the solid, the line X′ Y′ will represent the position of the line X Y at the end of the second. It is clear from the figure that if we remove the planes farther from each other by the distance O P plus Y P′, that is, twice O P, and assume that there is no slipping at the surface of the solids, then, since the relative positions of the successive layers are unaltered, the friction between contiguous layers of gas, and therefore the tangential stress on the solids will be the same as it is where there is slipping and where the planes are at the original distance X Y from each other.

That the distance O P is, as Maxwell showed, equal to μ/σ, is readily seen. For, if we assume that the surfaces of the fixed and moving solids are alike, σ will be the same for both. If, also, we call the distance Y Y′, or X′ O, x, the tangential stress at either solid must be σx, since x is the velocity of C D relative to the gas in contact with it, and also the velocity of the gas in contact with A B, relative to A B. But the stress between consecutive layers is, since the distance between the planes is one centimeter, equal to $\mu\,(1-2x)$, and we have seen that this is the same as the stress at the solid. We have, then,

$$\sigma x = \mu\,(1 - 2x)$$

or,

$$x = \frac{\mu}{2\mu + \sigma}$$

or,

$$\sigma x = \frac{\sigma\mu}{2\mu + \sigma}$$

$$= \mu\left(\frac{1}{1 + 2\mu/\sigma}\right)$$

$$= \text{the force at the solid.}$$

But the form of this result shows that this is the force which must be applied to the solid, C D, to maintain a velocity of one centimeter per second if the distance from the fixed solid is $(2\,\mu/\sigma + 1)$ and there is

no slipping. If there is no slipping, then, in order that the stress at the solid should be the same as when there is slipping, namely σx, the distance between the planes must be increased by $2\ \mu/\sigma$, that is, twice the coefficient of slip as defined by Maxwell. In the figure, then, the distance O P is equal to μ/σ.

The investigation of the law of friction on a body at pressures smaller than those at which Maxwell worked was taken up by Kundt and Warburg,[9] but without adequate means of measuring pressure. They verified Maxwell's law, and concluded, both on theoretical grounds and from their experimental data, that over a certain range of pressure the coefficient of slip varies inversely as the density of the gas, and therefore, so long as Boyle's law holds, it varies inversely as the pressure. Sutherland [10] reached the same conclusion by a different method. If this is true and μ is constant, then σ, as defined above, must be proportional to the pressure. When the density of the gas has been so far reduced that the molecules seldom collide in passing from one solid surface to the other, then the friction is largely superficial or external, and one would expect that at less densities the resistance offered to the movement of the solid body would be well-nigh proportional to the pressure.[11]

Maxwell's viscosity apparatus consisted, essentially, of a disk of glass suspended with its plane horizontal and between two other larger fixed parallel disks of glass. The middle disk performed oscillations in its own plane. When the distance D, between the fixed and moving surfaces is small, Maxwell's formula for this apparatus becomes

$$\lambda - K = \frac{c\,\mu}{D} \tag{I}$$

Where λ is the total logarithmic decrement,

K is that due to the friction in the suspending fibre, and is a constant.

μ is the coefficient of viscosity of the gas.

D, the distance between fixed and moving surfaces,

c, a proportionality factor.

This is the formula only when slipping is not taken into account. When slipping must be considered, as has been said above, the fixed and moving surfaces may be considered removed from each other by a distance $2\ \beta$, where β is the coefficient of slip.

---

[9] Wied. Ann, **158**, 1876.

[10] Phil. Mag., [5], **43**, 1897.

[11] Kundt and Warburg, Wied. Ann., **158**, 1876.

In that case equation (I) becomes,

$$l - K = \frac{c\mu}{D + 2\beta} \tag{II}$$

where

l is the total decrement at a lower gas pressure, and the other quantities have the same significance as before.

Now, on dividing (I) by (II),

$$\frac{\lambda - K}{l - K} = \frac{D + 2\beta}{D}$$

$$= 1 + \frac{C}{p}$$

since β is inversely proportional to p. This equation may be written in the form

$$\left(\frac{\lambda - K}{l - K} - 1\right) p = C. \tag{III}$$

This equation is deduced by Sutherland; and it is also of the form of that used by Kundt and Warburg in their investigation to verify their theoretical conclusion that the coefficient of slip varies inversely as the pressure. As has been said, however, they were unable to measure the lower pressures, and so the law was not submitted to a test at very low pressures.

The relation between pressure and logarithmic decrement expressed here is the one utilized by Sutherland[12] to determine p after the constants in the foregoing equation have been determined. The data for this purpose he found in Crookes' paper on 'Viscosity of Gases at very High Exhaustions.' There the logarithmic decrement of a vane of mica, suspended with its plane vertical, and performing oscillations about a vertical diameter, is given for pressures ranging from that of an atmosphere to 0.02 millionths of an atmosphere. The pressure was measured by a McLeod gauge. Sutherland's formula applied to these results gave very good agreement from about 0.2 mm. to about 0.01 mm. in the pressure as measured by the gauge, and the pressure as calculated from the formula. Below 0.01 mm. the agreement was not good.

Stokes[13] has shown that, in this apparatus, the coefficient of viscosity is not proportional to the logarithmic decrement, and therefore, as

---

[12] Phil. Mag., [5], **43**, 1897.

[13] Note added to Crookes' paper.

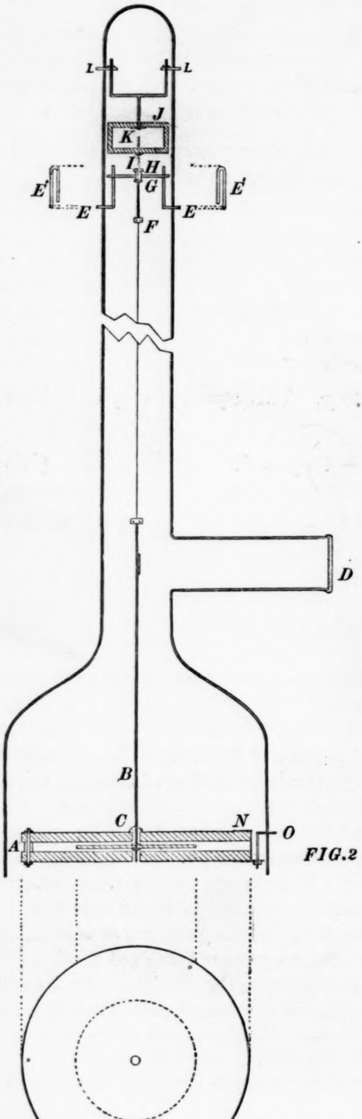


FIG. 2

Sutherland has pointed out, a fairer test of the correctness of (III) might be made with a viscosity apparatus like that for which the formula was deduced.

This suggestion has been followed in the construction of the instrument whose description follows.

Viscosity Apparatus — Description. — The viscosity apparatus used in these experiments consists of two circular glass plates 8.2 cm. in diameter and 0.3 cm. thick (A, Figure 2). They are pierced by a hole in the centre 0.3 cm. in diameter, and by three holes near the edge, equally spaced, and 0.15 cm. in diameter. Through the latter are passed small threaded platinum bolts, provided with nuts, for the purpose of binding the plates together. In order that the latter may be fixed at a certain distance apart, they are separated by three short glass tubes, through the bore of each of which passes one of the aforesaid bolts. These glass tubes are ground to the same length, 0.35 cm.

These are the two fixed plates in Maxwell's apparatus, while between them, concentric with them, and parallel to them, is suspended the vibrating disk, which consists of a plate of glass 4.4 cm. in diameter and 0.09 cm. thick. It has a hole in

the centre about 0.075 cm. in diameter, through which passes the closely fitting supporting wire, B, Figure 2. The latter is provided with a hard-soldered shoulder, against which the plate is clamped by means of the nut below. Platinum foil washers are placed between the glass and the nut. On the supporting wire at C are fixed three feet, long enough so that, if the swinging plate were lowered, they would reach the upper surface of the upper plate just before the suspended plate could touch the lower plate. This is a measure of safety, as will be seen when the process of making and suspending is described. At the proper height for the window, D, is fastened a mirror of platinized glass. It is supported on the wire by thin platinum foil, which is fused to the wire, and then folded around the edges of the mirror. At the end of the wire is a clamp, consisting of two flat pieces of platinum 1.25 mm. thick, 3 mm. wide, and 6 mm. long. One is hard-soldered to the wire so that one surface is in the axis of the wire. The other is fastened upon this by two screws. The fibre is fastened at the top in the similar clamp, F, at the end of the wire, I. The nut, H, into which the wire, I, is screwed, carries a cross-bar, G, the ends of which are engaged by the platinum wire hooks, E and E, shown detached at E′ and E′. There is also a check-nut, K, on the wire, I, above H, which can be screwed down upon H to hold the clamp and cross-bar in any desired relative position. The upper end of the wire, I, screws into a nut of the same material as itself on the soft iron armature, J, which is supported by the swivel-head, K. The supporting cross-bar is itself fastened to the two vertical threaded pieces passing through the supports, L L, and furnished with nuts for support and adjustment.

As the telescope and scale were to be used in observing, and as the whole apparatus was to be heated, a plane parallel window, D, which would bear a temperature of 300° C., must be secured. It was decided to close in the end of a glass tube, and grind the inside and outside flat and parallel to each other, and then seal the short horizontal tube at the proper place to the long vertical one. By using a large lump of optical glass, and using the oxygen flame, it was found possible to close the end with glass of fairly uniform density. In ordinary glass tubes, the streaks in the glass cause difficulty, and besides, almost invariably a small sort of pit is formed at the centre in the process of sealing. At this point, the density of the glass is different from that of the rounded part, thus giving an irregular lens effect. The method of grinding is to use a ring tool with which to make a groove around the inside just as large as the size of the tube will allow. The groove allows clearance for the emery when the process of grinding flat is

undertaken. A thoroughly satisfactory window is obtained in this way.[14]

After the glass vessel has been hermetically sealed, the soft iron armature, J, can be turned by means of a magnet fixed suitably outside. The magnet is placed on a circular platform of brass surrounding the tube so that the poles of the magnet control the armature within. The platform is supported on the top of the box containing the glass part of the apparatus. The platform has a circular groove in it which a circular brass ring fastened to the lower side of the magnet exactly fits, so that the magnet may be slid around without altering its position with respect to the tube.

To put this apparatus together required some care in handling, and no little skill in glass-blowing.[15] The large cylinder of glass was blown about 40 cm. long, and when the three supporting wires, one of which is shown at O, had been sealed in, the lower end of the cylinder was opened. To the upper end the narrower tube was joined, and the short tube bearing the window, D, was then sealed on at the proper place. The next step was to fasten the fibre so that it might be heated to a temperature of, say, 300° C., without danger of slipping or breaking. Various attempts were made. For example, the end of the fibre was platinized, and electroplated with copper to the supporting wire, — a very troublesome operation. It was found, moreover, that when heat was applied a break occurred where the quartz came in contact with the metal, so that this method was abandoned. Carbon cement was also tried, but discarded owing to the uncertainty whether all of the volatile substances which it contains were driven off. Clamping was resorted to, and the simple form of clamp shown in the figure adopted. With the clamp, the danger of snapping the fibre just at the edge of the metal is considerable. To minimize this, platinum foil was wrapped around the part of the fibre to be placed in the clamp. The upper large plate was now pushed on to the wire, B, and the small disk clamped to the wire in the manner already described. The lower plate was then placed on a support, the glass separating pieces placed temporarily upon it, and the disk suspended in position between the plates. A short range horizontal telescope was then focussed on the edge of the suspended disk, so that, when the latter was made to revolve, one could tell very readily if the wire, B, had been fixed at right angles to the disk. In order to remedy any want of perpendicu-

[14] This work was done by Mr. Lundin of the Alvan Clark Optical Company, Cambridge.

[15] Mr. Oelling, the glass worker of the Jefferson Physical Laboratory, has done the glass-blowing with great care and efficiency.

larity the upper plate and the disk were lifted together, and the wire carefully bent at a point close to the disk. After some trials it was found possible to adjust, so that the maximum difference in height of the two ends of a diameter of the disk was less than 0.07 mm.[16] The width of the disk was 44 mm. The large plates were then fastened together as described above, and the three clamps, one of which is shown at N, were placed so as to fit the supporting wires, one of which is seen at O.

Suspending cords were now placed temporarily around the glass separating tubes and the plates raised by them, in an inverted position, allowing the armature and its belongings to be suspended by the fibre. The whole was then carefully lowered into the enclosing vessel, which had been placed temporarily bottom upward. When the plates were in position on the supporting wires, the various wires of the suspended part were put in place by means of a tool made for the purpose, the various nuts turned on, and the whole turned into its natural position. The disk was raised by turning the armature, J, and the plates then made parallel to it. This adjustment was made by turning the supporting nuts on the wires O, O, O. Finally, the check-nut, K, being loose, the upper clamp was turned in the cylindrical nut, H, until the mirror faced the window, D. The check-nut was then tightened, and the suspended part lowered so as to rest on the tripod at C. The instrument could now be handled with only moderate care. The upper and lower ends were then sealed off, and it remained only to join this to the other parts of the apparatus.

The Transpiration Problem.

The first experiments on Thermal Transpiration were made by Feddersen,[17] but the full investigation of the phenomenon is due to Professor O. Reynolds.[18] He showed that, if the two surfaces of a plate of porous material which divides a mass of gas into two separate portions are kept at different temperatures, the gas will force itself through the channels in the material from the cold surface to the hot until a certain difference of pressure is reached. In the case of air, he found it possible to establish, in this way, as much as 6 mm. of mercury differ-

[16] Stokes has shown that a small inaccuracy in this adjustment involves a rather large error in the measurement of the resistance encountered by the moving disk. If the layer of air between the latter and the fixed disk is wedge-shaped, considerable energy is used up in crowding the gas between the fixed and moving disks. The adjustment attained here is sufficiently accurate to avoid this difficulty.

[17] Pogg, **148**, 302 (1873).

[18] Phil. Trans., **170**, 1880.

ence of pressure between the two sides of the plate, where the mean pressure in the gas was 760 mm. His experiments established the fundamental law that the difference of pressure reached is increased by increasing the difference of temperature between the surfaces of the porous plate, and that, given a certain difference of temperature, the amount of the gas transpiring, or the height to which the difference of pressure will grow, is governed by the relation between the mean free path of the molecules and the diameter of the conducting space. When, at a given pressure, the diameter of this space is diminished, the difference of pressure attainable is increased. Since this is true, it is readily seen that, when the transpiration spaces are large, it is only necessary to reduce the density of the gas to get the same results as those obtained at greater densities with spaces of capillary dimensions.

Sutherland[19] renews the theoretical discussion of this phenomenon, and by a different method arrives at substantially the same result. His result appears in an expression relating the difference of pressure to the mean pressure, the coefficient of viscosity, the diameter of the transpiration tube, and the mean velocity of the molecules. He then considers the case of a circular vane of badly conducting material, placed in a circular space, which it fits rather closely, and, assuming that in the transpiration space thus formed the temperature of the gas is controlled by that of the vane and the surrounding annulus, he deduces an expression for the force which will tend to push the vane out of the plane of the annulus, when one side of the vane and the corresponding side of the annulus are heated. His result is expressed thus:

$$F = \frac{c}{Ap + B + 1/p} \qquad \text{(IV)}$$

where F is the force on the vane in arbitrary measure, and p the mean pressure expressed in, say, millimeters of mercury.

Sutherland has submitted his equation, with a considerable degree of success, to the experimental test, using data accumulated by Crookes in his work, on viscosity of gases at high exhaustions. Crookes measured not only the logarithmic decrement of the vane of mica mentioned above, but also the angle through which the vane was deflected when the light from a candle was made to fall on the blackened half of one of its surfaces. By measuring F and the pressure for three different pressures, the constants in (IV) can be determined, and then,

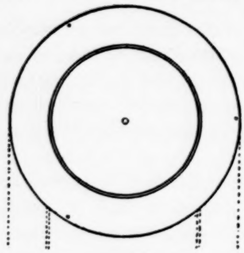
[19] Phil. Mag., **42**, 1896.

by measuring F, p can be calculated for any other degree of exhaustion and the result compared with the value of p obtained by the McLeod gauge. If (IV) is the proper relation between F and p, the values of the latter obtained from (IV) and from the gauge should be the same over any range of pressures where the results obtained by the gauge are reliable.

In Sutherland's hands, this formula has stood the test fairly well, but it must be added that the experimental conditions from which the data used by him were obtained did not well conform to the mathematical conditions assumed in the deduction of the above formula. Still, the results obtained by him indicate the direction in which to look for the full solution of the problem.

Transpiration Apparatus — Description. — Figure 3 is a horizontal section of the instrument through the centre of the suspended vane. The essential parts of it are a cylindrical glass vessel, A, Figure 3, about 7.5 cm. in diameter, in which is fixed an annulus of mica 1.25 cm. wide, the plane of the annulus being a cross section of the vessel. The vane, suspended in a manner to be described later, will pass through the opening of the annulus, leaving about 0.75 mm. clearance. The vane is 4.7 cm. in diameter, and is less than 0.1 mm. in thickness. The vane is clamped between a shoulder and nut at B, Figure 3. The supporting wire, C, is in turn fastened to another, D, by collar and check-nut, U and V. D bears a mirror, E, and F and G are counterpoises. At K is a short metal tube, shown in vertical section at H, Figure 3*a*. The wire, I, Figure 3*a*, is furnished with shoulder and check-nut, seen at L and J, and just fits the tube, H. The fastening of the fibre is the same here as in the viscosity apparatus.

In Figure 4 is shown the arrangement by means of which the suspended system can be raised or lowered without orienting, and oriented without raising or lowering. A is a soft iron armature, D a supporting swivel-head, the rod of which presses through guide-pieces, E and F, and terminates in a second supporting swivel-head, G. To the vertical rod is fixed a cross-bar, H, furnished with vertical guide-pieces, I I, which pass through the wire loops, J J. To the armature, A, the fibre clamp is fixed. The clamp is kept in the centre of the tube by means of the wire, K, which has a loop through which the clamp wire, L, just passes. M is another soft iron armature supported, as shown, on a screw passing through a nut, N, which, in order to avoid complications from inequalities of expansion, is made of the same material as the screw.



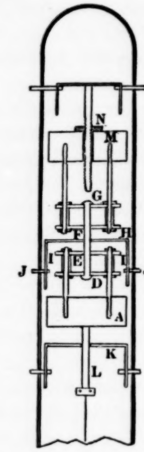


FIG. 4

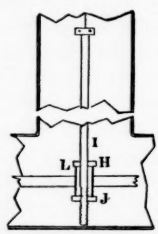


FIG. 3a

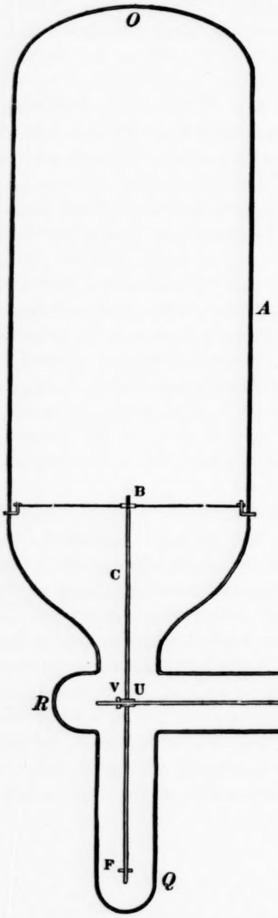
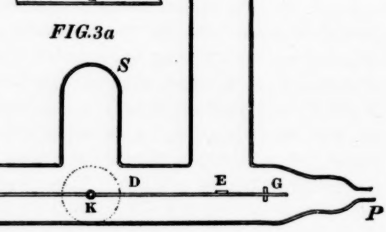



FIG. 3

By turning M alone, the whole suspended system is raised or lowered; while by turning A alone, a twist is given to the fibre while the height of the system is unchanged. Magnets, similar to the one used for turning the armature in the viscosity apparatus, and mounted so that they are entirely free from the glass part of the apparatus, are used to turn the armatures A and M.

Openings were left at O, P, Q, R, S, and at a point directly beneath the suspending fibre, until the annulus and vane with its supporting wire, C, were put in place, the wire D, bearing mirror and counterpoise, screwed into the collar U, and the adjusting apparatus bearing the fibre and the wire I placed in position. When J had been tightened, the whole suspension was raised by turning M. This operation was carried out in order to determine whether the wire D had been screwed just far enough into the collar U so that, when the system was raised, the plane of the vane might be parallel to that of the annulus. To remedy any want of parallelism, the suspended system was lowered again, and the collar U turned with respect to D. After going through the process of adjusting many times, the vane was finally placed in the proper adjustment, which was maintained by means of the nut, V.

In order to avoid breaking the fibre, during the process of closing the various openings in the apparatus, the suspension was lowered to resting loops not shown in the figure. All of the openings mentioned above were now closed save P, to which the connecting tubes shown in Figure 5 were joined.

The McLeod Gauges. — Two McLeod gauges of the form shown at A, Figure 5, were used. They have different factors, one of them being suited for the measurement of comparatively high pressures, while the other, the highest factor of which is about 69,000, is suited for the highest vacua. At B B are air-traps to prevent small bubbles of air, which may have adhered to the glass tube below, from slowly rising and destroying the vacuum. As it was necessary to keep the mercury in the gauge for some time and to keep it clean, the ordinary method of raising the mercury by raising a reservoir of it attached by rubber tubing to the barometer tube of the gauge, was replaced by the method which Figure 5 will explain. Here are two reservoirs C and D, attached to their respective gauges. The entrance to each is fitted with a stopcock, and the single leading tube has a three-way cock at E. F is an air-tight reservoir, S a pipe leading to the water tap, and T a waste tap. Water rises through S and forces air from the top of F against the mercury surfaces in C and D.

Arrangement of the Apparatus as a whole. — Figure 5 shows how these three pieces of apparatus are connected. It will be seen that the viscosity apparatus and the transpiration apparatus are joined by the glass tube L, and that a common tube leads from them to the pump by way of a tube, M, containing granular silver, and one, N, containing sulphur which has been fused and then powdered. The sulphur is intended to prevent mercury vapor from passing from the parts of the apparatus in which there is mercury to the other parts. The silver is to absorb the sulphur vapor. As a means of testing the purity of the air used, a spectrum tube, not shown in Figure 5, is inserted between the tube containing silver and the main part of the apparatus. The figure shows how the McLeod gauges are connected to the other parts of the apparatus. The tube, P, leads to the pump. The connection between the viscosity apparatus and the transpiration instrument is a long spiral tube, so that the former may be rotated through a considerable angle without disturbing the latter.

The piece marked A′ is for the purpose of admitting dry air or other gases, and consists of about 2.5 m. of 2.5 cm. tubing containing phosphoric anhydride, and 1.25 m. containing chloride of calcium. The gas is admitted through a tube, G, whose end passes under the flared out end of the barometer tube, H. The bottle, I, contains mercury forming a seal. The drying tubes may be connected to a bellows, if air is to be experimented with, or to a gas generator. The gas rises in bubbles through the mercury to the bulb, J. The small bent tube K is to prevent the mercury from being driven through the apparatus when a bubble of gas rises.

The mercury pump has no stop-cocks. It has one mercury-sealed valve. The auxiliary is a mechanical pump which can reduce the pressure to two or three millimeters.

The viscosity apparatus, the transpiration apparatus, and the gauges, are placed on an iron support inside of an electric oven. The first and second pieces are each placed in a double walled sheet iron box, whose wall space is packed with asbestos. The fronts of these boxes are removable, as is also the front of the oven. The purpose of the oven is to provide means of heating the whole apparatus to a high temperature to insure drying and to free the inside of the glass from the carbon dioxide which invariably adheres at ordinary temperatures and pressures.

When the different parts of the apparatus were set in position and sealed together, and before the suspensions had been raised, or any pumping had been done, the front of the oven was closed and heat applied so as to maintain the temperature at 200° C. during a whole

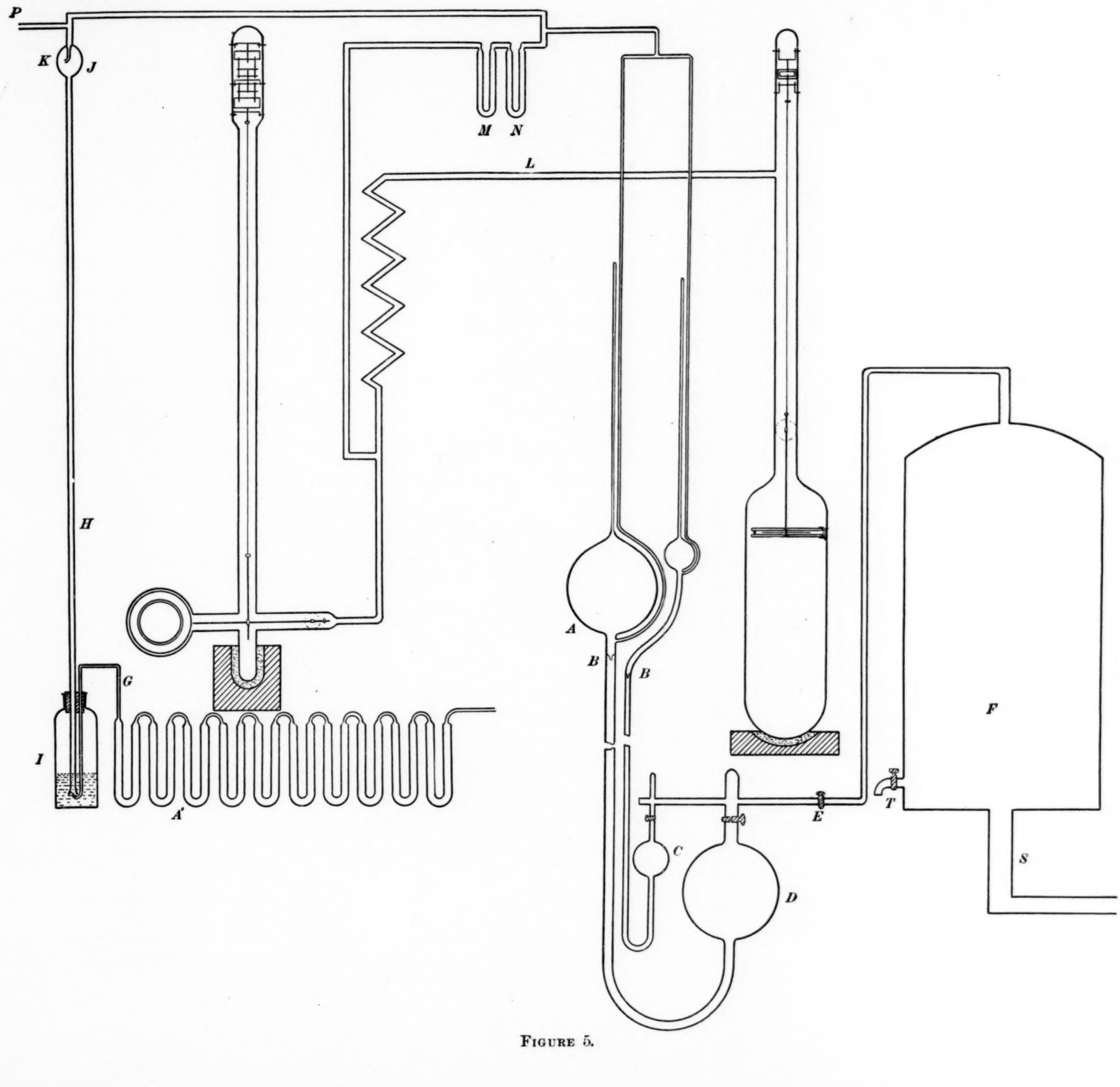


FIGURE 5.

day. There was no cracking of the glass, so that one was assured that the joints had all been well annealed.

Drying Process.

This process consisted in first pumping out the air until a pressure of a few millimeters of mercury was reached, then forcing in through the tube G a large quantity of dry air. This was pumped out again, and more dry air then forced in. This process was repeated many times. After this preliminary drying, and when the pressure was low, the front of the oven was closed and a current turned on sufficient to raise the temperature to 200° C. This temperature was maintained for several hours, and during this time the pump was kept going, and for about one hour the connecting tubes were kept hot by a Bunsen flame.

Perhaps it may be well to state here, that in the effort to distinguish between a real leak in the apparatus and what will produce a similar effect, viz., the slow "evaporation" of the gas from the inner surface of the vessel, it was found expedient to remove the sulphur tube and silver tube. Under these conditions the pumping can be done much more quickly. When the heating and pumping process above described had been resorted to, and after the oven had been allowed to cool down to room temperature, it was found that a vacuum of about 0.0003 mm., as indicated by the gauge, had been reached. When, however, the apparatus was simply allowed to stand the pressure greatly diminished, and a condition was finally reached when a slight rise in the temperature of the room would cause the pressure to increase greatly, much more than could be accounted for by the temperature coefficient. Then again a small decrease in temperature, say four or five degrees, would cause a corresponding anomalous decrease in the pressure. In one case where the temperature of the room reached 14° C. and remained there for over a day, the pressure was less than one third of what it was when the temperature had reached 18° C. for the first time after the oven had been allowed to cool down. This is mentioned merely to show how important a part the temperature of the glass plays in determining what the vacuum in the apparatus at any time is.[20]

After the complete drying process the spectrum showed that the moisture had been removed, and the vessel was then filled with dry air.

The suspensions were then raised to the proper height by turning

---

[20] After it had been proved that there was no leak in the apparatus the sulphur and silver tubes were replaced.

the magnet which controls the armature, A, Figure 2, and M, Figure 4. A cathetometer was used to determine when the swinging disk in the viscosity apparatus was midway between the other two. As the mica vane could not be well viewed with the cathetometer, its proper adjustment was judged by its symmetry of position with respect to the surrounding ring of mica.

Method of Experiment.

Viscosity Apparatus. — This instrument is held in position at the top by a snugly fitting collar fixed in the top of the enclosing box. It is fixed at the bottom, as shown in Figure 5, so that by giving the arm, L, a slight, slow angular movement, the disk can be set rotating without giving it a serious pendulous motion. This is especially true at the higher pressures, but more care in starting is necessary when the gas becomes rarer. About three quarters of an hour's training is always given to the fibre before observations begin. After allowing everything to become steady, the position of rest for every swing is observed, and when the damping is rapid about seven or eight swings is all one can get for any one start given to the disk. Sufficient accuracy is obtained by using the results got from starting the disk four times. This applies only to the work at the higher pressures; as will be seen, the plan is changed when the density diminishes, for then many more arcs can be obtained before the amplitude of swing becomes too small.

In getting the mean decrement, the logarithms of the fifth, sixth, seventh, and eighth arcs are taken respectively from the first, second, third, and fourth, and the result in each case divided by four. At the lower pressures, where many more arcs can be obtained in a series, and where as a consequence the error in observing any one is increased, the eleventh is taken from the first, the twelfth from the second, and so on, and the results divided each by ten. The decrement used is then got by taking the mean of some sixteen such decrements.

Transpiration Apparatus. The source of light used for the purpose of illuminating the blackened face of the mica vane and annulus was a twenty-five candle power incandescent lamp, suited to a voltage of forty. The current used was that from a storage battery.

The zero position of this instrument is that position of the armature, M, Figure 4, which allows the vane to hang in the plane of the annulus, when there is no irradiation of any part of the apparatus. To read this position, there is a pointer attached to the magnet which controls the armature. The pointer coincides with a radius of a graduated circle placed on the upper magnet platform, concentric with the tube

in which the fibre is. Great care is taken in setting to turn the magnet in one direction so that any movement which the magnet, and therefore the pointer, may make, which is not made by the armature, may be eliminated. It was found, however, that with an ordinary permanent magnet this elimination was not satisfactory, so that a strong electro-magnet was finally used. This proved quite satisfactory, and certainly the error here is not so great as that arising from the innate difficulty of setting the vane to the proper position. To determine when the vane is in the proper plane a telescope is focussed on a scale reflected in the mirror, E, and the point on the scale which corresponds to the proper position of the vane is noted. This, of course, remains the same so long as the disposition of the apparatus is unchanged.

From what has been said previously on the relation between the force on the vane and the size of the transpiration space, it follows that for any given pressure the force on the vane in this instrument will be greatest when the vane and annulus are in the same plane, for then the annular space is least. The object is to measure the force when this disposition is secured. Since the blackened surface is turned towards the heat source, when this surface is irradiated the vane recedes, because pressure grows on the hot side. By turning the magnet, enough torsion is given to the fibre to bring the vane forward again into its zero position. If too much torsion is given, and the vane is thrown in the least past the annulus, then it continues to swing forward, because the force on it diminishes as its distance from the annulus increases. A glance through the telescope shows when this has happened, and in this case the magnet must be turned back and another trial be made. Much practice is required that one may set the vane, even with moderate success. The difference between the readings on the circular scale when the vane is and when it is not irradiated is the torsion necessary. As is allowable, the force on the vane is assumed to be proportional to the angle of torsion when the latter is not large.

In this maximum effect we have a method of finding the zero position without actually looking to see that the vane and annulus are co-planar. The method consists in illuminating the vane, setting to the maximum point, and noting the mark indicated by the cross-hair of the telescope. This mark indicates the zero position. It is really best to determine the two balancing points for any angle of torsion, one when the vane is behind the annulus and the other when it is in front. As the torsion on the fibre is increased the places of balance, indicated by the cross-hair of the telescope, approach each other,

and the mean of these locates with sufficient accuracy the maximum point.

So long as the shutter over the window of the containing box is closed, the zero position should remain unchanged. With ordinary jacketing, it was found that this was not the case. Indeed, in some cases there was as much variation as 25°. This could be due to one or all of three causes, viz.: convection currents, unequal heating of the bulb or indeed any change of temperature of the apparatus, or to small charges of electricity on the vane or containing vessel.

To guard against the first of these, as has been said, the apparatus was enclosed in a double-walled box with an asbestos interspace. But since the McLeod gauge, which, of course, must be left uncovered, and is connected with this instrument through various obstructed tubes, responds quickly to any change of temperature in the room, any change in this temperature will cause a flow of gas either towards the transpiration instrument or away from it. To overcome this difficulty, a house whose walls are of asbestos and double, with an air interspace, was built surrounding the apparatus. In it were placed four electric heaters, the current through which can be made and broken by a relay worked by a battery circuit, in which is a thermostatic strip. In this way the temperature of the whole space — about 24 cubic meters — enclosed can be maintained very nearly constant. The temperature was kept near 22° C. day and night, so that one might begin work at any time and know that the apparatus was not far from a uniform temperature.

The second cause, that of temperature change in the instrument itself, proved the most serious. Before the constant temperature room was resorted to, invariably during the day with rising temperature there was an advance in the zero position, i. e., the apparatus acted as if the vane were illuminated, while during the night, as the temperature fell, the contrary was the case. The blackened surface would, of course, absorb heat which was being conducted in through the walls of the containing vessel faster than the clear surface, and a transpiration would begin which would have the same direction as that arising from the illumination of the vane. When the rise of temperature ceased, then the two surfaces would gradually reach the same temperature, and in the absence of cause for transpiration the vane could be put into its zero position without using force. Only by enclosing the bulb of the apparatus in a pretty thick (1.5 mm.) brass shell (the object of which is to distribute quickly the heat which enters from without) and then packing the box full of cotton wool, leaving only a channel to the mirror, E, closing the door of the box, and then packing it all around

with wool, was it possible, even with the help of the constant temperature room, to remove serious disturbances arising from this cause. It was also necessary to prevent, as much as possible, light from shining on any part of the apparatus, as this would produce unequal heating. The light from the telescope lamp passed through a water window, and was allowed to shine only when observations were being made.

The trouble arising from electrical charges can readily be distinguished from those just mentioned, for, with it, the vane tends to take up a fixed position, and force is required to change that position in either direction. The force on the vane arising from unequal heating is unidirectional. Trouble from electrical charges showed itself most distressingly after an attempt to wrap the bulb in tin foil for the purpose of minimizing outside heat effects. The friction between the metal and the glass produced a very pronounced charge. It was noticed also that after filling the vessel with air in the manner already described the suspension, or rather the whole apparatus, had become so charged that its attraction for a pith-ball could easily be detected. A rather weak radioactive substance was placed within the brass shield as near as possible to the vane, and after several days the charge had disappeared. During the time when the box was being packed with the wool the brass shield was kept to earth, and it has remained so connected.

By attending carefully to all of these points it was found possible to control the vane so that after the current was turned on in the lamp and before the shutter was opened there was no great change in the zero reading.

Since the object is to establish a relation between pressure and the difference of pressure maintained between the ends of a transpiration space, and since any surface along which there is a variation of temperature must give rise to transpiration, it is clear that if the entire bulb of the instrument is illuminated, there will be various causes contributing to the total transpiration pressure, because the different parts of the surface of the bulb will not be equally heated. Now Sutherland has developed equation (IV) on the assumption that the only transpiration space contributing to the effect is annular in form. This is the simplest form of space to deal with analytically, and the attempt to establish the desired relation is likely to prove more successful with such a space than with an irregularly shaped surface unequally heated. In the form of the apparatus finally adopted the bulb is of a somewhat different form from that shown in Figure 3. The mica ring is placed against the conical posterior part of the glass vessel, so that the whole of the vane and ring, together with that narrow part of the glass in contact

with the ring, may be irradiated. Thus the gas will transpire past not only the inner edge of the ring, but also the outer edge, and yet the whole bulb is not heated. There are then really two concentric annuli, and no space by which difference of pressure can be effaced except through the transpiration spaces themselves.

Before packing the box with wool, the lamp was placed at its proper distance, about 80 cm., and in such a position that the light from it fell perpendicularly upon the vane. Then a bright metal diaphragm was placed on the glass opening in the door of the box just large enough so that the base of the cone of light from the lamp would cover the desired area. The only part of the bulb illuminated, other than the mere ring contiguous to the outer edge of the mica ring, was, then, that immediately within the box and opposite the hole in the diaphragm, so that the heated portion of the bulb was at least 16 cm. away from the vane. The spot of glass irradiated was only about 4 cm. in diameter. To minimize the heating of this by absorption, the light was made to pass through about 2.5 cm. of glass and about 7 cm. of water. The water was kept running from one bottle to another through a water window placed between the lamp and the apparatus. The temperature of the water suffered no perceptible change in the course of an experiment which lasted some three hours.

Perhaps the greatest difficulty experienced in the mere handling of this apparatus is that caused by the great inertia of the suspended system. It is easily seen that the least jar given to the apparatus in turning the control magnet may give to the system a momentum which is easily many times greater than the force which it is desired to measure. To make conditions as favorable as possible, the platform on which the magnet was placed was finally fastened so that it and the magnet were entirely free from the glass vessel and from the containing box; but the more or less jerky motion of the armature as it moves under the influence of the magnet still gives the same trouble, though in a much less degree. This difficulty is greatly increased at low pressure; for when the density is small, the viscosity of the air is so small that the vane, though large, is a very inefficient damper, and so when the suspended system suffers a slight disturbance it moves freely until it strikes one side of the containing vessel; then it rebounds, strikes the opposite side, and so on. It is true that the suspended system might have been made lighter than it is, but, at best, it must be rather heavy, for the form which it must assume in order that it may be put together is such as to preclude its being made as light as it should be for convenience.

It is clear also that the equilibrating torsion given to the fibre will

depend upon how long the light is allowed to shine on the vane, because the vane, after it is heated, becomes a radiator, and the surrounding vessel is heated by radiation from the vane. This increase in the force goes on for some hours, and in that time there is a chance for very inconvenient changes in the temperature of the vessel and vane due to extraneous sources of heat, e. g. the different distribution of the heat in the room when the lamp is lighted, and when the experimenter is in the room. The source of light was enclosed in a double-walled asbestos chimney, which communicated at the top with a flue, but the sides of this chimney became heated in time. It was found best to set the illuminated vane as accurately as possible (a process taking from a half to three quarters of an hour), and then close the shutter and set again. The difference is taken as the torsion necessary. The results given below were obtained in this way. It was found that the zero position of the vane, i. e., the position of equilibrium after the shutter had been closed, did not change as much as 5° in an hour. At this point, however, an error is introduced which tends to make the result too large. The maximum angle of torsion obtained was 250°, so that the error here considered is not very serious for such an angle of torsion; but as the pressure is diminished the angle diminishes, and at pressures where the angle of torsion has fallen to, say, 25°, this error becomes serious.

McLeod Gauges. — In reading the gauges, special attention was paid to these points:

First, the mercury, when lowered, was never allowed to sink much below the point of connection between the volume tube and the pressure tube. The reason for taking this precaution is that the narrow bore of the latter offers considerable resistance to the passage of gas into it, and thus when the mercury is raised after having been allowed to sink well down, more than the right quantity of gas is thrown up into the bulb, and the pressure will read too high.[21] Still further to ensure accuracy on this point, the mercury is made to rise very slowly indeed until it has passed the entrance to the pressure tube.

Second, during the ascent of the mercury the bulb and tubes are tapped repeatedly; and in setting on any mark, care is taken to tap both tubes until one is certain that the mercury has really settled down.

Third, in the measurement of each pressure the mercury is raised twice, and, for each time, generally one measurement is taken at each

[21] Ramsay and Baly mention the necessity of taking this precaution. Phil. Mag., [v], **38**, 1894.

of two marks on the volume tube of the gauge. The cathetometer is used to learn when the setting is right, and to measure the mercury column.

Of course, the nearer the top of the volume tube one works, the more serious the error made in setting becomes. Indeed, it is better not to use a mark which is less than, say, 2 cm. from the top.

Results.

The following table gives the results obtained so far. In the first column is given the gas pressure as measured by the McLeod gauge. The pressure is expressed in millimeters of mercury. The gas employed in this case was air. In the second column the values of the logarithmic decrement in the viscosity apparatus corresponding to these pressures are given, and in the third the amount of torsion given to the fibre of the transpiration instrument at the various pressures to balance the force due to gas action on the vane. The significance of the fourth and fifth columns will appear later.

The table shows how slowly the logarithmic decrement decreases at first as p is diminished. That there is an appreciable diminution in l even when the pressure is great, e. g. 20 cm., is due to the fact that the distance between the fixed and moving surfaces in the apparatus is very small, and therefore the coefficient of slip, β, is comparable with it. To put the matter in another way, l is large because of the small distance between the plates, so that small relative changes in it can be perceived. Yet the table shows that at a pressure of 5.8 cm. the resistance to the moving disk is only one half per cent smaller than it is at atmospheric pressure.

With falling pressure the rate of decrease of l increases, and for pressures less than 0.01 mm. l and p become approximately proportional. At such pressures the mean free path of the molecules of air is over half a centimeter in length, and is therefore several times the distance between the fixed and moving disks, so that the friction is largely superficial. Figure 6 shows the relation between l and p over the range from where $p = 0.530$ mm. to where $p = 0.00085$ mm. A unit on the axis of abscissas represents a pressure of 0.01 mm., and a unit on the axis of ordinates represents $l = 0.00333$.

The resistance which the disk meets is due to at least two causes, and we shall see that there is some evidence that there is a third. There is first the friction of the air on the disk, and second, the friction in the suspending fibre. The former diminishes with decreasing density of air, but the latter is a constant provided that the temperature of the fibre is maintained constant, as was the case in

these experiments. The change in the temperature of the apparatus from day to day was rarely as much as 0°.5 C., and was generally much less than this during the course of one experiment.

The proportionality already referred to would evidently be more exact if the constant part of the resistance, i. e., that due to the imperfect elasticity of the fibre itself, could be determined and taken from the total resistance. An effort has been made to determine what is the resistance due to the air in the vessel alone. The method of

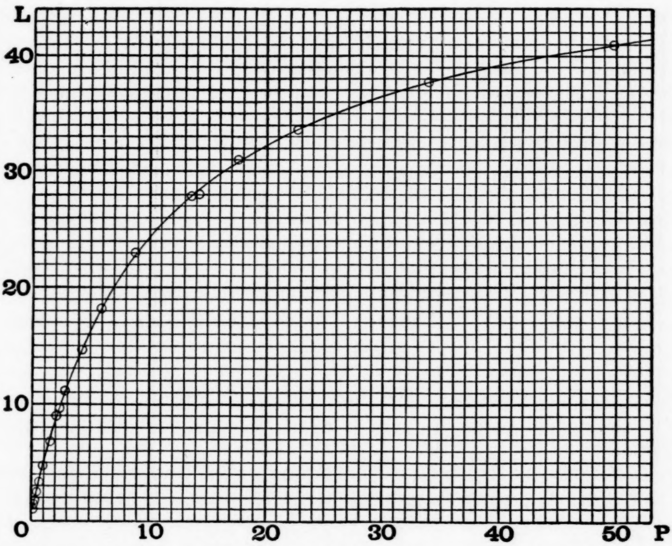


FIGURE 6. The curve shows the relation between pressure and logarithmic decrement over the range of pressure, $p = 0.530$ mm. to $p = 0.00085$ mm. The unit of pressure is 0.01 mm. The unit of logarithmic decrement is 0.00333.

procedure was to pump out the air to a certain low pressure, measure the decrement, then pump to a still lower pressure, measure the decrement again, and so on. If the exhaustion be carried far enough, a limiting value for l should be approached. This did, indeed, happen, but the limiting value seemed larger than one would have expected it to be; for it would seem that the friction in such a suspending fibre should be exceedingly small. The following results show how the decrement and the pressure diminished as the pumping proceeded. Pump began at 10.30 A.M., when the pressure was about 0.001 mm.

TABLE I.

p	l	R	C	p, cal. from l.	p, cal. from R.
mm. 761.5	0.1655				
514.1	0.1652				
200.8	0.1650				
58.4	0.1648				
1.42	0.1533	62°			
0.824	0.1461	122°	0.1112		
0.498	0.1364	170°	0.1076		
0.339	0.1258	190°	0.1085		
0.226	0.1121	202°	0.1096		
0.176	0.1032		0.1082		
0.142	0.0938		0.1108		
0.135	0.0931	248°	0.1072		
0.0880	0.0765	240°	0.1050	0.0913	
0.0590	0.0607	210°	0.1051	0.0611	
0.0425	0.0491	170°	0.1048	0.0442	
0.0284	0.0370	143°	0.1040	0.0297	0.033, or 0.41
0.0232	0.0324	115°	0.1012	0.0249	0.025, or 0.54
0.0211	0.0300	115°	0.1017	0.0226	0.025, or 0.54
0.0148	0.0228	70°	0.1011	0.0159	0.015, or 0.93
0.00928	0.01580	50°	0.1002	0.0101	0.010, or 1.32
0.00595	0.01113		0.1000	0.00649	
0.00410	0.00837		0.1002	0.00446	
0.00261	0.00607		0.1007	0.00282	
0.00177	0.00450		0.1113	0.00173	
0.00186	0.00478		0.1053	0.00193	
0.00085	0.00332		0.0999	0.00093	
0.00024	0.00229		0.1119	0.00023	

Decrement at 11.30 A.M. was 0.00234
" " 12.30 P.M. " 0.00234
" " 4.45 P.M. " 0.00228

In this time there had been about four hours of continuous pumping. According to the gauge the pressure at the final stage was 0.00019 mm. It is possible, however, that the pressure was not completely equalized through the apparatus, although the time allowed to elapse between successive strokes of the pump was sufficient to insure that the equalization at this time must have been nearly complete. At 9.30 P.M. the pressure had become 0.00024 mm. and the decrement at the same time was 0.00229. No pumping had been done since the last measurements were made at 4.45 P.M. At 12.10 P.M. on the next day the pressure was 0.00027 mm., while the decrement was 0.00236. The limiting value of the decrement seems to be not much less than 0.00230.

It is probable that this limiting value of l is due not only to the friction in the fibre, but also to the friction of mercury vapor on the disk. It was noticed that in the spectrum, after the current had been allowed to run for some time, the mercury lines gradually appeared, as if there were some slight deposit of mercury on the walls of the tube which slowly evaporated under the heat due to the current. This would indicate the possibility of a like deposit on the inner surface of the other parts of the apparatus, for it will be remembered that the spectrum tube is placed between the tube of silver and the main part of the apparatus.

The fact that the limiting value of l obtained is larger than had been anticipated would be explained by the presence of mercury vapor. Its presence would add a third factor to the resistance experienced by the disk, and one which would diminish certainly while pumping was in progress, but which would increase again as evaporation of the mercury, or the diffusion of its vapor to the viscosity apparatus from the other parts of the apparatus, proceeded.

Proceeding for the moment on the assumption that the decrement due to the air is proportional to the air pressure in the apparatus where the pressure is small, we can deduce the value of that part of the decrement due to friction in the fibre and the friction of the mercury vapor combined. For if m represents this constant part of the decrement we have,

$$p_1 : (l_1 - m) :: p_2 : (l_2 - m)$$

Also, for corresponding values for p and l, we find from Table I,

$$p_1 = 0.00177 \text{ mm.} \qquad l_1 = 0.00450$$
$$p_2 = 0.00024 \text{ mm.} \qquad l_2 = 0.00229$$

Using these values, we find for m the value 0.00194. If now this quantity m be inserted for K in the equation relating p and l, as given above, viz.:

$$\left(\frac{\lambda - K}{l - K} - 1\right) p = C$$

C can be determined for the different pressures, or C having been determined for the higher pressures, for example for those between 1 mm. and 0.1 mm., the values of p corresponding to the lower values of l can be solved for. The value of λ used in this process was 0.1655, and the value of C used was the mean of the first seven values given in the fourth column of Table I. The fourth and fifth columns of Table I give these results.

A comparison of the numbers given in the fifth column with those in the first show that the values of p deduced from the observed values of l are in general greater than the observed values of p.

It must be admitted at once that the foregoing method of getting the value of K is a very imperfect one, and it is inserted only tentatively. It is now proposed to remove the sulphur and silver tubes and place a vessel containing liquid air so as to surround a portion of the tubes connecting the pump with the apparatus, so that not only may all vapor be removed, but also that the very highest possible vacuum may be reached. The decrement will then be measured. This should give the value of the part of the decrement due to friction in the fibre. The liquid air will then be replaced by liquid carbon dioxide, which will remove the vapor but not the gas to be experimented with. It is hoped that this method of procedure will settle the only point that seems to remain in doubt in this part of the investigation. The full discussion of the law relating l and p is reserved until this step has been taken.

With regard to the results given in Table I for the transpiration instrument it must be stated that the smaller numbers in the third column may quite easily have an error of ten per cent. Figure 7 shows the results graphically. In this figure, a unit on the axis of abscissas corresponds to 0.01 mm. of pressure, while on the other axis a unit represents 10° of torsion. That portion of the curve which corresponds to pressures below those for which the torsion is a maximum, approaches a straight line, and it is apparently a line which passes very nearly through the origin. It is perhaps allowable to assume that it does go through the origin; for the force here, unlike the friction in the

other apparatus, depends entirely on the gaseous contents of the vessel. There can be no force when these have all been removed. If there is any vapor present, however, there will still remain some force when the air has been pumped out. As we have seen, there are indications that mercury vapor is present, but in small quantity, and at the lowest pressure at which the transpiration apparatus was used the torsion necessary to balance the force on the vane, were the contents of the vessel the vapor alone, could not be more than 5°. This is readily

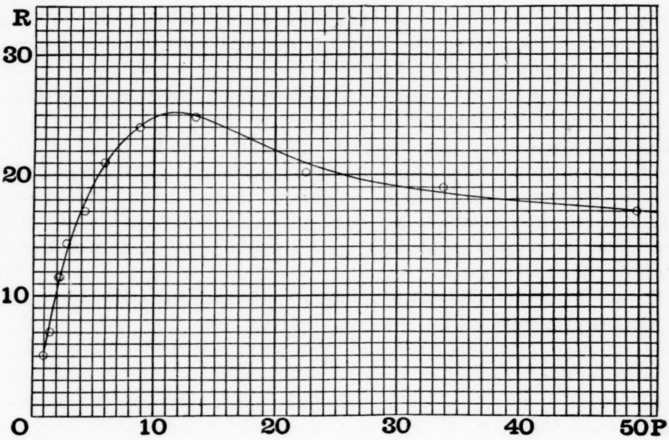


FIGURE 7. The curve shows the relation between pressure and force due to gas action on a circular vane over the range of pressure, $p = 0.498$ mm. to $p = 0.0093$ mm. The unit of pressure is 0.01 mm. The unit on the axis of ordinates represents an angle of torsion of 10° given to the fibre supporting the vane. This torsion is proportional to the force due to gas action on the vane.

seen by considering the torsion at these small pressures to be proportional to the pressure.

If we confine our attention to the part of the curve from the origin up to the maximum point, and use the larger values of the torsion from which to determine the constants in Sutherland's equation, viz. :

$$F = \frac{c}{A\,p + B + 1/p} \qquad \text{(IV)}$$

we get $A = 0.00187$, $B = 0.0152$, and $C = 25.67$. In the computations 0.005 mm. has been taken as the unit of pressure and 1° as the

unit of torsion. If now we insert these numbers in the above equation and solve for the values of p, which correspond to the various values of the torsion given in the third column of the table, the results given in the last column are obtained. The form of the equation shows that there are two values of p for any particular value of the torsion. If we differentiate (IV) with respect to p to get the value of p, for which the torsion is a maximum, we get $p^2 = 1/A$, or $p = 23.1$. With the millimeter as the unit this is the same as $p = 0.116$ mm. The value of the torsion for this value of p is 254°. The curve indicates a maximum where $p = 0.120$ mm., and its value is 252°. The further discussion of equation (IV) is reserved until more and more accurate data have been collected.

With the object of rendering the instrument more easily handled, and thus making it susceptible of greater accuracy, it is proposed now to modify the form of the suspended system in order to reduce its moment of inertia. When this has been done the results from this instrument should be quite as reliable as those from the other, and with the experience which has been gained both in constructing and handling the apparatus it seems quite certain that the desired object will soon be accomplished.

Grateful acknowledgment is made to Professor Trowbridge for placing at my disposal all the resources of the laboratory, including the services of a mechanician[22] and of a glass-blower.[23] Without the assistance rendered by them the apparatus could not have been constructed. To Professor Hall, who first called my attention to the problem, I am indebted for much advice and encouragement.

JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY.

---

[22] Mr. Thompson of the Physical Laboratory.
[23] Mr. Oelling.